Indian Institute of Engineering Science and Technology, Shibpur

Department of Mathematics

Subject: Mathematical Analysis

Subject Code: MA5101

Full Marks: 100   Total Load/week-4,   Credit-4

Functions of several variables. Differentiability of mappings $f: R^n \rightarrow R^m$. Derivative as a linear transformation. Role of Jacobian. Derivative of composite mappings, Contraction mapping principle, Implicit function theorem, Inverse function theorem, Rank of a mapping, Rank theorem.

Differential forms, k-forms in R^n and their properties, Theory of integration on differential form, Stoke's Theorem.

Measurable space. Construction of measurable sets and measure through outer measure. Almost everywhere properties of functions. Measurable functions and their integration. Finite measures. Consideration of Lebesgue integral on R^n, Fatau's lemma monotone convergence theorem, dominated convergence theorem. Convergence in measure.

Reference books:

1. Bruckner A. M., Bruckner, J. B., Thomson B.S., Real Analysis, Prentice Hall Inc, NJ, 1997.
2. Rudin W., Principles of Mathematical Analysis, McGraw-Hill, 1976.
3. Royden H.L., Real Analysis, MACMILLAN, 1963.

Subject: Complex Analysis and Numerical Analysis

Subject Code: MA 5102

Full Marks: 100 Total Load/week -4 credit -4

Complex Analysis (3 classes /week):

Topological structure of Complex Plane, Simply connected and multiply connected domains. Homotopic version.Spherical representation of extended complex plane, Analytic Functions, Harmonic functions, Subharmonic functions and applications, Littlewood's condition for subharmonic functions, Complex integration, Cauchy's Theorem and Integral Formula, Winding Numbers, Cauchy's estimate, Morera's theorem, Liouville's theorem, Fundamental theorem of Algebra. Maximum modulus principle, Schwarz Lemma, Taylor series, Laurent series, Zeros and poles of complex functions, Meromorphic functions. Hurwitz Theorem, Classification of singularities, Residue theorem, Argument Principle, Theorems of Rouche and Gauss-Lucas, Contour integration and its applications to improper integrals, Evaluation of real integrals, Improper integrals involving sines and cosines, Definite integrals involving sines and cosines, Integration through branch cut, Conformal Mapping, Mobius Transformations, Schwarz-Christoffel Transformation. Weirstrass' theorem, Montel's theorem and its application to establish Vitali's theorem. Harnack's inequality and its use in establishing Harnack's principle.

Numerical Analysis (1 class/week):

Eigenvalues and Eigenvectors of real matrix : Power method for extreme eigenvalues and related eigenvectors, Jacobi's and Householders methods for symmetric matrices.

Spline Interpolation : Cubic Spline.

Approximation of Functions : Least squares polynomial approximation, Approximation with orthogonal polynomials, Chebyshev polynomial, Lanczos economization.

Numerical Integration : Newton-Cotes' formula of closed type, Gaussian Quadrature.

Numerical solution of initial value problems for Ordinary Differential Equation (ODE) : Multistep Predictor-Corrector methods, Adams-Bashforth method, Adams-Moulton method, Milne's method, Convergence and Stability.

Two-point boundary value problems for ODE : Finite difference and Shooting methods.

References

Complex Analysis:
1.Churchill, R. V. and Brown, J. W., "Complex Variables and Applications" 5th edition, McGrawHill. 1990.
2. Gamelin, T. W., "Complex Analysis", Springer-Verlag 2001.
3. Greene R., and Krantz, S. G., "Function Theory of One Complex Variable", 3rd Edition, GSM, Vol. 40, American Mathematical Society. 2006.
4. Lang, S., "Complex Analysis", Springer –Verlag, 2003.
5. Narasimhan, R. and Nivergelt, Y., "Complex Analysis in One Variable", Birkhauser, Boston, 2001.
6.Ahlfors, L. V., "Complex Analysis", 3rdEdn., McGrawHill, New York,1979.
7.Conway, J.B. "Functions of one complex variable", Springer –Verlag, 1978.

Numerical Analysis:
1. Introductory Methods of Numerical Analysis – S.S. Sastry (PHI Publications)
2. Introduction to Numerical Analysis – A. Gupta and S.C. Bose (Academic Press)
3. Numerical Methods for Scientists and Engineers – K. Sankara Rao (PHI Publications)
4. Numerical Analysis – N. Datta and R.N. Jana (Shreedhar Prakashani).

Subject : Linear Algebra and Abstract Algebra

Subject Code: MA5103

Full Marks: 100 Total Load/week-4 Credit-4

Linear Algebra (2 classes/week):

Quick review of Vector Space and Linear Transformation.

Canonical Forms**:** Eigenvalues and eigenvectors of linear operators, Minimal polynomial, Cayley-Hamilton Theorem, Triangulation (Schur's theorem), Diagonalisation, Invariant subspaces, Generalized eigenvectors, Cyclic subspaces and annihilators, Rational canonical form,Jordan canonical representation.

Inner Product Space**:** Definition, Gram-Schmidt ortho-normalization, Orthogonal projections.
Linear operators and adjoints, Self - adjoint, unitary and normal operators, Spectral theorem for normal operators.

Bilinear forms**:** Symmetric and skew-symmetric bilinear forms, real quadratic forms, Sylvester's law of inertia, positive definiteness.

Abstract Algebra (2 classes/week):

Group Theory: Review of basic Group Theory, Homomorphism, Isomorphism of groups, First and second isomorphism theorems, automorphisms and automorphism group, Inner automorphisms, Direct and semi-direct products, Group Actions, Kernel and Stabilizer of Group Actions, Cayley's Theorem, Burnside theorem, Conjugacy classes, Class Equation, Cauchy's theorem on finite groups, p-group, Centre of p-groups, Sylow's Theorem and applications: simplicity of groups, Classification of finite Abelian groups, Finite groups, Structure theorem for finite Abelian groups, Normal and Subnormal series, Composition series, Jordan–Holder theorem, Solvable groups and Nilpotent groups.

Ring Theory: Review of basic Ring Theory, Properties of Ideals, Prime and Maximal Ideals, Two-sided ideals and Quotient Rings, Euclidean Domain, Euclidean Algorithm, Principal Ideal Domain, Euclidean Domain is a Principal Ideal Domain, UFD, PID implies UFD, Polynomial Rings, Irreducibility of Polynomials, Eisenstein's criterion of irreducibility

References:

Linear Algebra**:**

1. Linear Algebra by Stephen H Friedberg, Insel and Spence.
2. K. Hoffman and R. Kunze: Linear Algebra, 2nd Edition, Prentice Hall of India, 2005.

Abstract Algebra :

1. Abstract Algebra by David S. Summit & Richard M. Foote,
John Wiley & Sons
2. Fundamentals of Abstract Algebra by D.S.Malik, John N. Mordeson & M.K.Sen, The McGraw-Hill Companies, Inc.
3. Abstract Algebra by I.N. Herstein, Prentice-Hall

Subject: Computational Techniques and Programming

Subject Code: MA5121

(Departmental Elective-I/1)

Full Marks: 100 Total Load/week -3 credit -3

Computational Techniques :

Finite Difference Technique
Numerical solution of different types of Partial Differential Equations by Finite Difference method
Explicit and Implicit Methods, Crank Nicholson Method
Formulation and analysis of control equations of neural, ecological and social networks.
Flowchart Algorithm

Programming :

Identifiers and keywords, Constants, Variables, Arithmetic Operators, Library Functions, Input/ Output Statements.
Relational Operators, Logical Operators Bitwise Operators, Unary Operators, If-Else Statement, Switch statement, Goto statement and Label.
Iteration statements: For Loop, While Loop, Do Loop, Nested Loop, Continue and Break statements.
Array and Structures : Declaration, Concept of One Dimensional and Multi Dimensional arrays, Defining Structure, Declaration of Structure Variable, Accessing Structure members, Nesting of structures, Array of structures.
"C" function, User defined and Library functions, Prototype of function, Call by Value, Pointer Variable, Call by Reference, Nesting of functions, Recursion, Array as function-Argument.

Data Structure: Linked list, Stack, Queue. Binary Tree.

References:
Numerical Analysis – K. Atkinson & W. Cheney
Numerical Analysis and mathematics of scientific Computing - David Kinciad & Ward Cheney,
Introductory Method of Numerical Analysis – S.S. Sastry
The C Programming Language – Brian W. Kernighan, Dennis M. Ritchie
Let Us C – Yashavant Kanetkar
Programming in ANSI C – E. Balagurusamy

Subject: Probability and Statistical Methods (Some Selected Topics)

Subject Code: MA 5122

(Departmental Elective-I/2)

Full Marks: 100 Total Load/week -3 credit -3

Borel-Cantelli lemmas, Helly-Bray theorem, Glivenko-Cantelli theorem, Scheffe's theorem, Polya's theorem, Skorokhod's representation. Characteristic functions- ***Lévy's*** continuity theorem, Applications- Central Limit Theorem (Lindeberg- ***Lévy***), Lyapunov's CLT.

Sufficient and complete statistics, Minimal sufficient statistics, MVUE, MLE and its optimality properties, Neyman-Fisher factorization criterion, Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao Inequality.

A brief introduction to Resampling Plans - the Jackknife and Bootstrap techniques.

Introduction to MCMC methods- Importance sampling, Gibbs sampler, The Metropolis-Hastings algorithm.

References:

1. Grimmett, G. and Stirzaker, D. *Probability and Random Processes*. 3rd edition, Oxford University Press.
2. Bickel, Peter J. and Doksum. *Mathematical Statistics: Basic Ideas and Selected Topics*, Volume I, Second Edition.

3.Efron, B. The Jackknife, the Bootstrap and Other Resampling Plans (CBMS-NSF Regional Conference Series in Applied Mathematics).

Subject: Discrete Mathematics

Subject Code: MA5161

(Open elective I/1)

Full Marks: 100 Total Load/week -3 credit -3

Prerequisite: NIL

Set, Relations and Functions:

Set Theory: Concept of set, subset, super set, equality of sets, union and Intersection of sets, differences of sets, differences of sets, complement, power set, Cartesian product of sets, Venn diagram.

Relation: Binary relations-reflexivity, symmetry, anti symmetry, transitivity, equivalence, partition of a set.

Mapping: Injective, surjective, bijective mapping, identity and inverse mappings, composition of mappings.

Algebraic Structures:

Algebraic system, monoids, free monoids, Definition of Group, finite Group, Subgroup, cosets, Lagrange's theorem, Homomorphisms, isomorphisms and automorphisms. Permutation groups, Normal subgroup and Factor group theory in coding theory.

Definition of Ring, ring of integers, ring of integers modulo p (p being any prime or non-prime nos.), subring, ideals, module, finite module - example and properties, Submodule, ring homomorphisms, Polynomial rings, integral domain, field, finite integral domain as a field, finite fields.

Boolean Algebra:

Posets, lattices, principle of duality, Boolean algebra and its properties, Atoms, Disjunctive and Conjunctive normal forms, Application to switching algebra.

Mathematical Logic:

Proposition and definition of symbols, Truth tables, conjunction, Negation, Implication, Double implication, Tautology, Tautological equivalence, Proofs in propositional calculus, Conditional conclusions and indirect proofs, Truth set, Principle of induction and its variations.

Predicate calculus: Predicates, the statement function, variables and quantifiers, Predicate formulae, free and bound variables, valid formula and equivalence.

Graph Theory:

Elementary and basic concepts of graph theory, path and circuits, Euler graph, Hamiltonian path and circuits, Trees and fundamental circuits, Cut sets and its application, Planner and dual graph, Vector spaces of a graph, Matrix representation of a graph, directed graph, Generating functions, Partition of integers, Recurrence relations.

Text/Reference books

1. K. H. Rosen, Discrete Mathematics and its Applications, 6^{th} Ed., Tata McGraw-Hill, 2007.
2. N. Deo, Graph Theory, Prentice Hall of India, 1974.
3. Seymour Lipschutz, M. Lipson, Discrete Mathematics, Tata McGraw Hill, 2005.

Ordinary Differential Equations and Dynamical Systems

(MA 5201)

Full Marks: 100 Total Load/week-4 Credit-4

Group A: Ordinary Differential Equations (70 Marks)-3classes/week

Existence and uniqueness of solutions of initial value problems involving first order ordinary differential equations (ODEs). Sturm-Liouville's boundary value problems, characteristic numbers and characteristic functions, Orthogonal properties of characteristic functions, Orthonormal systems, Inhomogeneous boundary value problems: Green's function and its properties.General homogeneous linear ODE in the complex domain, Ordinary points and Singular points, Fundamental existence and uniqueness theorem, Analytic continuation of the solution, Fundamental sets of solutions, Wronskian, Series solutions near ordinary point and regular singularity point, Method of Frobenius, Sufficient condition for regular solution, Contour integral solution, Fuchsian type ODEs.
Hypergeometric Equation, Solution in Series: Hypergeometric Function, Riemann P- equation.
Legendre Equation, Solution in Series: Legendre functions of first kind and second kind, Legendre's polynomials, Different representations of Legendre's polynomials: Rodrigue's formula, Laplace's integral, Generating function for Legendre polynomial, properties of Legendre polynomials, Contour integral solution, Recurrence formulae, Associated Legendre equation.
Bessel Equation, Solution in Series: Bessel functions of first kind and second kind, Contour integral solution: Hankel function, Generating function for Bessel function of integral order, Recurrence formulae, Modified Bessel equation.

Group B: Dynamical Systems (30 Marks)-1 Class/week

Introduction to dynamical systems, Discrete & continuous dynamical system, Flows, Map,Evolution, Fixed points, Linearization of general systems, Eigen value-Eigen vector method, Phase plane analysis, Local and global stability analysis, Periodic solutions (Poincare Theorem, Bendixson's negative criterion, Dulac's criterion), Limit cycles (Poincare –Bendixson Theorem), Bifurcation in one and two dimensional systems, Introduction to Chaos.

References:

1. Ordinary Differential Equations by E. L. Ince

2. An Introductions to Ordinary Differential Equations by Earl A. Coddington

3. Special Functions and their Applications: N.N. Lebedev

4. Special Functions by W. W. Bell

5. Nonlinear dynamics and chaos with applications to physics, biology, chemistry, and engineering by Strogatz, S.H., 2018. CRC Press.

6. Differential Equations and Dynamical Systems by Lawrence Perko, Springer

Partial Differential Equations and Calculus of Variations

(MA 5202)

Full Marks: 100 Total Load/week-4 Credit-4

Group A: Partial Differential Equations (70 marks)- 3 Classes/Week

Introduction and formation of partial differential equations, General information about variety of solutions of PDEs, Solution of first order linear, Quasi-linear, semi linear and nonlinear PDEs, Cauchy's method of Characteristics, Compatible systems of first order PDEs, Charpit's Method and Special types of first order PDEs.

Solution of higher order PDEs with constant coefficients, Construction of Green's function with the help of delta function.
Linear partial differential equations of second order with variable coefficients, classifications, reduction to Canonical forms, Characteristic curves, adjoint and self-adjoint operators, Riemann's method.
Elliptic differential equations: Derivation of Laplace and Poisson equations, Properties of Harmonic functions, Separation of variables, Dirichlet and Neumann problems for rectangle and circle.
Parabolic differential equations: Occurrence of the diffusion equation, Elementary solution of diffusion equation, Solution of diffusion equation in Cylindrical and Spherical coordinates.
Hyperbolic differential equations: Occurrence of the wave equation, Solution of one-dimensional wave equation, Uniqueness of the solution of wave equation.
Green's function: Green's function for Laplace equation, wave equation, diffusion equation.

Group B: Calculus of Variations (30 marks)- 1 Class/Week

Introduction, Euler-Lagrange equation for fixed end points, Euler-Poisson equation for functional involving derivatives of higher order, Ostrogradsky equation for functional involving functions of several independent variables, Geodesics, constraints and Lagrange's multipliers, Hamilton's principle, Lagrange's equations, generalized dynamical entities, constraints of dynamical systems.
Euler-Lagrange equation for variable end conditions, transversality condition,
Rayleigh-Ritz method, Galerkin method for approximate solution.

References:

1. Introduction to Partial Differential Equations, 2nd Edition by K. Sankara Rao, Prentice- Hall of India.
2. Partial Differential Equations by Phoolan Prasad & Renuka Ravindram, New Age International Publishers
3. Linear Partial Differential Equations for Scientist and Engineers by TynMyint-U, Lokenath Debnath, Springer.

4. Elements of Partial differential equations by I. N. Sneddon, Dover
5. Introduction to Calculus of Variation by C. Fox
6. Calculus of Variation by A. S. Gupta, New Age.
7. Introduction to Calculus of Variation by Hans Sagan

Topology and Tensor Analysis
(MA 5203)

Full Marks: 100 Total Load/week-4 Credit-4

Group A: Topology (70 marks)- 3 Classes/Week

Basis of a topology. Continuous functions. Metric topology. Product topology. Box topology. Quotient topology. Compact topological spaces. Compactness in metric topology. Connected topological spaces. Local connectedness. Path connectedness and its relation to connectedness. First and second countable topological spaces. Separation axioms-Hausdorff, regular and normal spaces.

Group B: Tensor Analysis (30 marks) – 1 Class/Week

Tensor Algebra: n-dimensional space, Transformation of co-ordinates, summation convention, Kronecker delta, scalar/invariant, covariant and contravariant vectors, covariant, contravariant and mixed tensors, e-system, ε-tensors. Addition and subtraction of tensors, multiplication of tensors, symmetric and skew-symmetric tensors, Quotient law, conjugate tensor.Tensor Calculus: Riemannian space, Line element and metric tensor, conjugate metric tensor, associated tensor, magnitude of a vector, angle between vectors, Christoffel’s symbols, transformation of Christoffel’s symbols, covariant derivatives of vectors and tensors. Riemann-Christoffel tensor, properties, Bianchi’s identity, Ricci tensor, Ricci scalar and Einstein’s tensor, flat and non-flat spaces. Curvilinear co-ordinates: co-ordinate curves and surfaces, line element, base system, reciprocal base system, physical properties.

Reference Books:
1. J. L. Kelley, General Topology, Van Nostrand, 1955.
2. G. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
3. J. Munkres, Topology - a First Course, Prentice-Hall, 1975.
4. Barry Spain, Tensor Calculus: A Concise Course, Dover Publications,2003.
5. David C. Kay, Schaum's Outline of Tensor Calculus, McGraw-Hill, 2011.
6. *U. C. De*, Absos Ali Shaikh and Joydeep Sengupta, Tensor Calculus, Narosa Publishing House, 2016.
7. Zafar Ahsan, Tensor Analysis with Applications, Anshan Ltd, 2008.
8. J. K. Goyal and K. P. Gupta, Tensor Calculus and Riemannian Geometry, Pragati Prakashan, 2021

Mathematical Biology
(MA5221)

(Departmental Elective - II/1)

Full Marks: 100 **Total Load/week-3** **Credit-3**

Continuous population model:

Single species models, Logistic model and its stability analysis.

Harvest in ecological models, Fishery models, Mathematical model of two interacting species. Lotka-Volterra Prey-Predator model, Lotka-Volterra competition model, Continuous model of three interacting species, Nutrient cycling, Oscillatory and chaotic system. Enzyme Kinetics.

Discrete population model:

Single species simple models. Density independent and dependent models. Cobweb method of solution, Discrete logistic model, Discrete Prey Predator model, Delay in Discrete model. Periodic solution and chaos in Discrete models.

References:

1. Deterministic Mathematical Models in Population Ecology, 1980, H. I. Freedman, Wiley publication.
2. Elements of Mathematical Ecology, 2001, Mark Kot, University press, Cambridge.
3. Mathematical Models in Biology, 2005, L. E. Keshet, SIAM Publication.
4. Mathematical Biology,2002, J.D. Murray, Springer Publication.

Mathematical Optimization Techniques
(MA5224)

(Departmental Elective-II/2)

Full Marks: 100 Total Load/week-3 Credit-3

Introduction to Optimization Techniques: Problem formulation, Solution convention.

Linear Programming: Duality and Sensitivity Analysis:

Complementary slackness conditions, Dual Simplex Method.

Discrete changes in the cost vector, Discrete changes in the requirement vector, Discrete changes in the coefficient matrix, Addition of a variable, Addition of a constraint.

Integer Programming:

Branch- and- Bound Algorithm, Cutting- Plane Algorithm.

Nonlinear Programming: Multivariable Optimization with Constraints

Method of Lagrange Multipliers, Kuhn-Tucker necessary and sufficient conditions of optimality.

Network Analysis

Scope and Definition of Network Models, Network Representation, Critical path computations for CPM, Critical path computations for PERT, Project time vs project cost.

Dynamic Programming:

Bellman's principle of optimality, Recursive relations, System with more than one constraint, Solution of LPP using Dynamic Programming.

References:

1. M.S. Bazaraa, H.D. Sherali, C.M. Shetty, Nonlinear Programming: Theory and Algorithms, 3rd Edition, John Wiley & Sons, 2006.
2. G. Hadley, Nonlinear and Dynamic Programming, Addison-Wesley, 1964.
3. T. C. Hu, Integer Programming and Network Flows, Addison-Wesley, 1970.
4. J.C. Pant, Introduction to Optimization: Operations Research, New Delhi, Jain Brothers, 2004.
5. H.A. Taha, Operations Research: An Introduction, Pearson Prentice Hall, 2007.

Advanced Optimization and its Applications

(MA 5261)

(Open Elective - II/1)

Full Marks – 100 Total Load/week-3 Credit- 3

Non-Linear Programming:

- Mathematical preliminaries

- **Unconstrained optimization techniques** – The golden search method, The Fibonacci search method, Steepest-Descent method, Newton's method, Davidon-Fletcher-Powell (DFP) method, Conjugate gradient method, Fletcher and Reeves conjugate gradient algorithm, Applications and some case study.
- **Constrained optimization techniques** – Lagrange's method, Multivariable optimization with inequality constraints, Kuhn-Tucker necessary and sufficient conditions, applications and related case studies.
- **Quadratic Programming Problem (QPP)** – Definition, Kuhn-Tucker condition for QPP, Solution of QPP by Wolfe's method, Duality in QPP, Applications in Portfolio analysis, Manufacturing design problem.
- **Information Theory** – Definition, Mathematical derivation of Information, Entropy, Shanon's Entropy and its properties, Description of Communication Channel, Channel capacity, Memoryless channel.
- **Fundamental concepts of Geometric programming:** Fundamental concepts of Geometric programming unconstrained posynomial geometric programming, degree of difficulty, constrained posynomial and signomial geometric programming. Some real-life applications on geometric programming.

References:

1. A first course on Operations Research and Information Theory – S. K. Mazumder. New Central Book Agency, Kolkata (First edition published in 2014, second edition is likely to be published on or before June, 2020).
2. Mathematical Programming – N. S. Kambo. East-West Press.
3. Operations Research – Kanti Swarup, P. K. Gupta, Man Mohan. Sultan Chand and Sons.
4. Geometric Programming for Design and Cost Optimization - Robert C. Creese. Morgan Claypool Publishers.

Department of Mathematics

Continuum Mechanics

(MA 6101)

Full Marks - 100 Total Load/week-4 Credit- 4

Group A : (50 marks) - 2 classes/week

Theory of strain : Deformation, Infinitesimal strain tensor, Geometrical interpretation of infinitesimal strain, Principal strains, Strain invariants, Compatibility equations.

Theory of stress : Forces, Stress vector, Stress tensor, Principal stresses, Stress invariants, Equilibrium equation.

Linear elastic materials, Constitutive equations in linearly elastic solids – Generalized Hooke's law, Strain energy function, Elastic symmetry – different types of materials.

Group B : (50 marks) - 2 classes/week

Introductory. Notions and Physical Properties of Fluid :

Definition and basic concept of fluid dynamics , Fundamental properties of fluids , The continuum hypothesis , Volume forces and surface forces acting on a fluid , Hydrodynamic pressure , Conditions at a boundary between two media , Euler's momentum theorem , D'Alembert's paradox , The flow past an obstacle.

Kinematics of Fluid Motion :

Introduction, Classifications of fluid flow, Specification of fluid motion :

(a) Lagrangian method

(b) Eulerian method

Velocity and acceleration of a fluid particle, Flux across any surface, Material, local and convective derivatives, Equation of continuity (Conservation of mass) :

(a) Euler's form

(b) Lagrange's form

Equivalence between Euler's and Lagrange's form, Symmetrical forms of equation of continuity:

(a) Cylindrical symmetry

(b) Spherical symmetry

Boundary conditions :

(a) Kinematical

(b) Physical

Condition for boundary surface, Patterns of flow lines :

(a) Path lines
(b) Stream lines
(c) Streak lines

Stream tube and stream filament, Angular velocity vector, Flow and circulation.

Equations Governing The Motion of a Fluid :

Introduction, Euler's equation of motion of an inviscid fluid, Equation of motion relative to moving axes, Euler's generalized momentum theorem, Bernoulli's equation, Pressure equation, Pressure equation referred to moving axes, Bernoulli's theorem, Applications of Bernoulli's theorem, Steady motion, Impulsive motion, Equations of motion under impulse:

(a) Vector form
(b) Cartesian form

The energy equation.

General Theory of Irrotational Motion :

Introduction, Some elementary definitions, Acyclic and cyclic irrotational motion, Kelvin's circulation theorem, Permanence of irrotational motion, Kinetic energy of finite and infinite liquid, Kelvin's minimum energy theorem, Mean value of the velocity potential over a spherical surface, Mean value of the velocity potential in a periphractic region, Some uniqueness theorems.

References :

1. I. S. Sokolnikoff : Mathematical Theory of Elasticity. McGraw Hill, 1956.
2. A. E. H. Love : A treatise on mathematical theory of Elasticity, McGraw Hill, 1954.
3. T. J. Chung, Applied continuum mechanics, Cambridge University Press.
4. D. S. Chandrasekharaiah and L.Debnath, Continuum Mechanics, Academic Press, 1994.

5. Gupta A. : Groundwork of mathematical fluid dynamics, Academic Publishers, 2013.
6. BatchelorG.K. : An Introduction to fluid dynamics , Cambridge University Press, 1967.
7. Frank M.White : Fluid mechanics. Tata Mc. Graw - Hill publishing company, New Delhi, 2008.
8. Goldstein S. : Modern developments in fluid dynamics , Oxford University Press, New York , 1938.
9. Lamb H. : Hydrodynamics , Dover Publications, New York , 1932.

10. MaseG.E. : Theory and problems of continuum mechanics , Schaum series.
11. McCormack P.D. and L. Crane : Physical fluid dynamics, Academic Press , New York, 1973.
12. Milne-Thomson L.M. : Theoretical hydrodynamics, The Macmillan Company , New York, 1950.
13. Streeter V.L. : Fluid dynamics, Mc.Graw Hill Book Company Inc. New York, 1948.
14. Streeter V.L. : Handbook of Fluid dynamics, Mc.Graw Hill Book Company Inc. New York ,1948.
15. Yuan S.W. : Foundations of fluid mechanics , Prentice-Hall of India pvt.Ltd. , New Delhi , 1969

Operations Research and Integral Transforms
(MA 6102)

Full Marks: 100 Total Load/week-4 Credit-4

Group –A: Operations Research (70 marks) - 3 classes/week

PART – 1:

Mixed integer programming problems- Definition, brief review of pure integer programming, Derivation of cutting plane constraints for mixed integer programming. Problems and related application in real life.

(1) **Parametric linear programming model** – Introduction parametric changes of cost vector (c), Parametric changes in Requirement vector (b), Problems and applications.

(2) **Queueing Models-** Introduction, Basic elements of queuing models, Poisson and exponential distribution in queueing model, Arrival process, Departure process, classification of queues, Model 1: $(M/M/1):(FCFS/\infty/\infty)$, Model 2: $(M/M/1):(FCFS/N/\infty)$, Model 3: $(M/M/C):(FCFS/\infty/\infty)$, Model 4: $(M/M/C):(FCFS/N/\infty; C<N)$, Model 5: $(M/M/C):(FCFS/N/N; C<N)$, Model 6: $(M/M/\infty):(FCFS/\infty/\infty)$; Problems and applications.

(3) **Dynamic Programming:** Introduction, Recursive equation of dynamic programming, subdivision problems in dynamic programming, characteristics of dynamic programming, problems and case study.

PART – 2:

(4) **Multi-objective linear programming:** Multi-objective linear programming by

(a) Weighted sum method
(b) E- constrained method
(c) Global criterion method
Some application of multi-objective linear programming problems.

(5) **Fundamental concepts of Geometric programming:** Fundamental concepts of Geometric programming unconstrained posynomial geometric programming, degree of difficulty, constrained posynomial and signomial geometric programming. Some real-life applications on geometric programming.

(6) **Replacement problem and system Reliability:** Replacement policy for items whose maximize costs increase with time money value is not counted.

Replacement policy for items whose maintenance costs increase with time and value of money also changes with time.
System Reliability: Failure rakes, bath-tub shaped failure rate, hazard rate, reliability systems in series and parallel arrangement.

(7) **Inventory control**: Introduction, types of inventory models, Model 1: Single item static model, Model 2: Model with no shortages and several production run of unequal length.
Model 3: Inventory model with shortages

Group –B : Integral Transforms (30 marks) - 1 class/week

Laplace Transform: Definition and properties, Class A function, Sufficient conditions for the existence of Laplace Transform, Laplace Transform of some elementary functions, Laplace transform of the derivatives, Inverse of Laplace transform, Convolution Theorem, Initial and final value theorems, Applications to ordinary and partial differential equations, Applications to Integral equation.

Fourier Transform: Fourier integral Theorem, Definition and properties, Fourier transforms of some elementary functions. Fourier sine and cosine transforms, Fourier transform of the derivative, Inverse of Fourier transforms, Convolution theorem, Applications.

References:

1. A first course on Operations Research and Information Theory- S. K. Majumder. New Central Book Agency, Kolkata (First edition published in 2014, Second edition likely to be published

in June 2020).
2. Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, - 2010.
3. F.S. Hillier and G.J. Lieberman : Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata McGraw Hill. 2010.
4. S. D. Sharma: Operations Research, Kedar Nath Ramnath & Co. 2012.
5. Mathematical Programming- N. S. Kambo. East-West Press.
6. Operations Research- Kanti Swarup, P. K. Gupta and Man Mohan. Sultan Chand and sons.
7. The Use of Integral Transforms - I. N. Sneddon.
8. Fourier Transforms - I. N. Sneddon.
9. Integral Transforms and Their Applications - B. Davies.
10. Integral Transforms and Their Applications - L. Debnath and D. Bhatta.

Functional Analysis and Integral Equation

(MA 6103)

Full Marks: 100 Total Load/week-4 Credit-4

Group –A : Functional Analysis (70 marks) - 3 classes/week

Normed and Banach Spaces: Definitions and elementary properties; example of some concrete Normed and Banach Spaces; Subspaces and Quotient Spaces.

Bounded Linear Operators: Definitions, examples and basic properties; Space of bounded linear operators; Finite dimensional normed spaces and compactness; Open mapping theorem and its consequences; Closed graph theorem and its consequences.

Bounded Linear Functionals: Definitions, examples and basic properties; Hahn-Banach theorem and its consequences.

The concept and Specific Geometry of Hilbert Spaces: Definitions and basic properties of Inner product and Hilbert spaces; Completion of Inner product spaces; Orthogonality of vectors and projection theorem.

Group – B : Integral Equations (30 marks) – 1 class/week

Linear integral equation of first and second kinds- Fredholm and Volterra type, homogeneous equation, eigen value and eigen function, separable kernel, solution of Fredholm integral equation with separable kernel, solution of Fredholm and Volterra integral equation by successive approximation, classical Fredholm theorems (Statement only), conversion of boundary and initial value problem to integral equation, conversion of integral equation to boundary and initial value problem, singular integral equation, solution of Abel integral equation.

References:

1. Functional Analysis by Balmohan Vishnu Limaye
2. Functional Analysis by P. K. Jain , O. P. Ahuja
3. Introduction to Functional Analysis with Applications, by A. H. Siddiqi, Khalil Ahmad, Pammy Manchanda
4. Linear Integral Equations by R.P. Kanwal
5. Integral Equations by F.G. Tricomi
6. A Course of Integral Equations by A.C. Pipkin
7. Applied Integral Equations by A.N. Chakraborty
8. Integral Equations and Boundary Value Problems by S.K. Pundir&R.Pundir
9. Integral Equations by Roy Singhania

Electrodynamics and Classical mechanics

(MA6201)

Full Marks - 100 Total Load/week-4 Credit- 4

Group A - Electrodynamics : (50 marks) - 02 classes/week

Electric and magnetic fields in space and inside matter. Biot-Savart law. Electromotive forces and electrical circuits. Maxwell's equations. Electromagnetic energy and momentum. Potential formulation of electromagnetic laws. Coulomb and Lorentz gauges. Electromagnetic waves. Electromagnetic radiation. Electromagnetic field tensor. Tensorial formulation of the Maxwell's equations. Relativistic electrodynamics and its Lorentz invariance.

Group B - Classical mechanics : (50 marks) - 02 classes/week

Holonomic and non-Holonomic systems. Constraints of motion. Lagrangian of a system. Lagrange's equations of motion with illustrations. Hamiltonian and Hamilton's equations of motion. The interpretation of Hamiltonian. Small oscillations and normal modes of oscillations. Canonical transformations. Generators of canonical transformations. Hamilton-Jacobi equation. Motion of a rigid body. Euler's theorem. Eulerian angles. Postulates of special relativity. Lorentz transformations. Transformations of velocities. Relativistic equations of motion.

Reference Books:

1. H. Goldstein, C. Poole and J. Safko, Classical Mechanics, Addison Wisley, 2000.

2. K.C. Gupta, Classical Mechanics of Particles and Rigid Bodies, New Age International Publishers, 1997.

3. B. K. Bagchi, Advanced Classical Mechanics, CRC Press, 2017.

4. D. J. Griffiths ,Intoduction to Electrodynamics, Cambridge University Press, 2017.

5. W. Greiner, Classical Electrodynamics, Springer, 1998.

6. J. D. Jackson, Classical Electrodynamics, John Wiley & Sons, 1962.

Waves and Vibrations in Elastic Solid Media

(MA 6221)

(Departmental Elective III/1)

Full Marks: 100 Total Load/week-3 Credit-3

Wave : Fundamental Waves : equivoluminal and dilatational waves in isotropic elastic solid medium, Body waves : P, SV and SH waves, Kinematical and Dynamical conditions for the motion of a surface of discontinuity, Velocity of waves in isotropic and anisotropic solid media, Wave surface, Plane wave, Surface wave : Rayleigh wave, Love wave.

Vibration : General theorem of vibration, Radial and rotatory vibrations of sphere, Torsional, longitudinal and flexural vibrations of circular cylinder.

References

1. I. S. Sokolnikoff : Mathematical Theory of Elasticity. McGraw Hill, 1956.
2. A. E. H. Love : A treatise on mathematical theory of Elasticity, McGraw Hill, 1954.
3. K. F. Graff : Wave Motion in Elastic Solids

ADVANCED FLUID DYNAMICS

(MA 6222)

(Departmental Elective III/2)

Full Marks: 100 Total Load/week-3 Credit-3

Vortex Motion :

Introduction , Some elementary definitions , Properties of vortex filament , Permanence of vorticity and vortex rings , Circular vortex , Pressure due a circular vortex , Ramkine's combined vortex , Rectilinear vortex , Single vortex filament , Two vortex filaments , Vortex pair , Vortex doublet , Source and vortex , Vortex filament parallel to a plane , Vortex filament parallel to two perpendicular planes , Vortex in and outside a circular cylinder , Vortex in the presence of a circular cylinder , Vortex sheet , Single infinite row of vortices , Two infinite rows of parallel vortices , Karman vortex street , The drag due to a vortex wake.

Liquid Surface Waves :

Introduction , General equation of wave motion , Mathematical representation of

(a) Progressive waves

(b) Stationary waves

Kinematical condition at the free surface , Pressure condition at the free surface

Surface waves :

(a) Progressive waves on the surface of a canal of finite depth

(b) Progressive waves on deep water

(c) Stationary waves on the surface of a canal of finite depth

(d) Stationary waves on deep water

Kinetic and Potential energy of progressive waves , Kinetic and Potential energy of stationary waves , Steady motion :

(a) Progressive waves on the surface of a canal of finite depth

(b) Progressive waves on deep water

(i) First order approximation to the wave speed

(ii) Second order approximation to the wave speed

(c) Progressive waves at an interface of two liquids

(d) Progressive waves at an interface of two liquids when upper surface is free

(e) Waves over a sinuous bottom.

Group velocity , Dynamical significance of group velocity , Capillary waves , Effect of capillarity on surface waves , Effect of capillarity on surface waves at an interface , Effect of wind on deep water , Long waves , Steady motion for long waves , Solitary waves.

General Theory of Stress and Rate of Strain :

Introduction , Stress across a plane at a point , Nature of stresses , Stress at a point , Symmetry of stress tensor , Stress in a fluid at rest and in motion ,Transformation of stress components:

(a) Two- dimensional stress components

(b) Three- dimensional stress components

Stress quadric , Principal stress , Principal planes , Fundamental theorem of stress , Mohr's circle diagram , Classifications of stress.

Strain analysis , Nature of strain , Transformation of rate of strain components :

(a) Two- dimensional rate of strain components

(b) Three- dimensional rate of strain components

Strain quadric , Fundamental theorem of strain , Classifications of strain , Relation between stress and rate of strain. Stoke'srelation , General motion of a fluid element.

Fundamental Equations of Viscous Fluid :

Introduction , Definition of viscosity , Effect of temperature and pressure on viscosity , Newton's law of viscosity , Types of fluids , Motion of a small body through a viscous medium. Stoke'slaw , The viscosity hypothesis , Boundary conditions in a viscous fluid , Navier-Stoke's equations of motion for viscous fluid , Conservation of momentum , The energy equation , Conservation of energy , Dissipation of energy due to viscosity , Diffusion of vorticity , Decay of vorticity , Circulation in a viscous fluid , Effect of viscosity on water waves in deep water.

References :

1. Gupta A. : Groundwork of mathematical fluid dynamics, Academic Publishers, 2013.
2. BatchelorG.K. : An Introduction to fluid dynamics , Cambridge University Press, 1967.
3. Frank M.White : Fluid mechanics. Tata Mc. Graw - Hill publishing company, New Delhi, 2008.
4. Goldstein S. : Modern developments in fluid dynamics , Oxford University Press, New York , 1938.
5. Lamb H. : Hydrodynamics , Dover Publications, New York , 1932.
6. MaseG.E. : Theory and problems of continuum mechanics , Schaum series.
7. McCormack P.D. and L. Crane : Physical fluid dynamics, Academic Press , New York, 1973.
8. Milne-Thomson L.M. : Theoretical hydrodynamics, The Macmillan Company , New York, 1950.
9. Streeter V.L. : Fluid dynamics, Mc.Graw Hill Book Company Inc. New York, 1948.
10. Streeter V.L. : Handbook of Fluid dynamics, Mc.Graw Hill Book Company Inc. New York ,1948.
11. Yuan S.W. : Foundations of fluid mechanics , Prentice-Hall of India pvt.Ltd. , New Delhi , 1969.